# Distributed Obstacle Avoidance-Formation Control of Mobile Robotic Network with Coordinated Group Stabilization

Nelson P.K. Chan, Bayu Jayawardhana, and Jacquelien M.A. Scherpen

*Abstract*—We present a distributed control law for a group of agents that solves the problem of formation control with obstacle avoidance and that can be combined with a coordinated group stabilization control law. In particular, we consider a control law that is given by a linear combination of distributed formation, distributed obstacle avoidance and centralized group motion control laws. Simulation results show the effectiveness of our proposed control law.

## I. INTRODUCTION

In this paper, we consider the problem of steering a group of agents as a formation towards a final desired destination while avoiding obstacles along the course of motion. A possible application can be found in smart manufacturing systems where a group of mobile robots are required to work together to transport an object from position A to position B. In such smart manufacturing systems there may be barriers which the mobile robots individually and as a group should avoid while carrying out the requested task.

In literature, numerous references can be found for achieving each of the individual tasks (formation control, group motion control, obstacle avoidance) or combinations thereof. See, for example [1] for an overview of approaches for achieving the formation control task based on the interconnection topology and sensing capability of the agents. In [2], [3], the weighted centroid tracking problem is considered in which the formation centroid is required to track an assigned task function. In [4], a decentralized controller is constructed based on the notion of *navigation function*. This controller guarantees the convergence of the multi-agent system to the desired formation while maintaining network connectivity and avoiding obstacles. In [5], the multi-agent collision avoidance problem is introduced and formulated as a nonlinear differential game. Dynamic feedback strategies are constructed guaranteeing the avoidance of collisions with obstacles or other agents while the individual agents reach their target. In [6], Lyapunov-like barrier functions are introduced such that the objective of avoiding obstacles can be composed together with other control objectives of the multi-agent systems into a single function for every

The authors are with Faculty of Science and Engineering, Engineering and Technology Institute Groningen, University of Groningen, Nijenborgh 4, 9747 AG Groningen, The Netherlands {n.p.k.chan, b.jayawardhana, j.m.a.scherpen}@rug.nl

agent. Furthermore, avoiding obstacles can be regarded as part of the requirements for guaranteeing safety of a (non)linear system; other requirements being the state and input constraints on the system. In [7], [8], the problem of synthesizing controllers for safety critical systems is considered. In both papers, the control design is based on the unification of both Control Lyapunov functions and Control Barrier functions for satisfying repectively the performance/stability properties and safety objectives.

In the current work, we propose a new control design for guaranteeing collision avoidance between the agents in formation and obstacles. Each individual agent is assumed to be able to sense the obstacle and action needs to be taken when the agent is within a certain threshold distance from the obstacle. The obstacle avoidance behavior of agent $i$ is then diffused towards the other agents in the network by means of a consensus type protocol explained later.

Compared with [5], our obstacle avoidance approach is decentralized, Moreover, in [5], the agents are not required to achieve a network objective (a desired formation or consensus) while moving to the individual target. Different from [4], we consider as well the movement of the formation as a whole towards a desired destination in the plane. This motion group control is for now a centralized approach.

The outline of this paper is as follows. In Section II, the problem of steering a group of agents towards a desired destination is formulated. The control laws for the different sub tasks are given in Section III. The merging of these control laws for an agent is as well considered. We illustrate our approach with an example in Section IV and Section V concludes the paper.

## II. PROBLEM FORMULATION

We consider a group of $N$ identical agents moving in the 2-dimensional plane. Each agent is modeled as a single integrator, i.e.,

$$\dot{p}_i = u_i, \quad i = 1, \dots, N, \tag{1}$$

where $p_i \in \mathbb{R}^2$ and $u_i \in \mathbb{R}^2$ denote the position and controlled velocity of agent $i$, respectively.

Without loss of generality, we consider obstacles that can be encapsulated by $m \geq 0$ circles in the plane where the centroid and radii of each circle $\mathcal{O}_k$, $k = 1, \dots m$, is given by $p_k^{\text{obs}} \in \mathbb{R}^2$ and $R_k^{\text{obs}} \in \mathbb{R}$, respectively. In this

case, the boundary of the $k$-th obstacle is defined by

$$\partial \mathcal{O}_k := \left\{ x \;\middle|\; \|x - p_k^{\text{obs}}\| - R_k^{\text{obs}} = 0 \right\}.$$

For our later description of distributed collision avoidance control law, we associate for each agent $i$ a parameter $R_i^{\text{safe}}$ which defines the safe distance to the boundary of any obstacle. Roughly speaking, when the agent's distance to the boundary of an obstacle is less than $R_i^{\text{safe}}$, then the distributed obstacle avoidance control will set in for the $i$-th agent. Throughout this paper, we will consider the case of $R_i^{\text{safe}}$ being constant for all $i$.

For the following definitions, let $\mathcal{P}_0$ denote the set of initial conditions which do not intersect $\mathcal{O}_k$ for all $k$.

*Definition 2.1 (local obstacle avoidance):* The $i$−th agent is said to *avoid collision with obstacle* $\mathcal{O}_k$ if for almost all $p_i(0) \in \mathcal{P}_0$ the trajectories $p_i(t)$ do not enter $\mathcal{O}_k$ for all $t \geq 0$, i.e., $p_i(t) \notin \mathcal{O}_k$ for all $t \geq 0$.

*Definition 2.2 (obstacle avoidance task):* The group of $N$ agents is said to achieve *obstacle avoidance* if every $i$-th agent avoids collision with all obstacles $\mathcal{O}_k$, $k = 1, \ldots m$.

The interaction between the agents is represented by an undirected graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$ with $\mathcal{V} = \{1, \ldots, N\}$ being the set of nodes representing the agents and $\mathcal{E} = \{(i,j) \in \mathcal{V} \times \mathcal{V}\}$ being the links/edges between the agents. The direct neighbors of each agent $i$ is denoted by $\mathcal{N}_i = \{j \mid (i,j) \in \mathcal{E}\}$. In the present work we assume that the agents can obtain relative position information from its neighbors, i.e., agent $i$ has access to $p_j - p_i$, $\forall j \in \mathcal{N}_i$. The desired relative position between the agents is encoded in a vector $p^* \in \mathbb{R}^{2N}$. Note that $p^*$ has to satisfy geometric constraints that define the shape of the formation. We refer to the exposition in [9], [10], [11] on the graph formalism of mobile robots' formation. For a given $p^*$, the group of $N$-agents is said to be in *the desired formation* if $p_j - p_i = p_j^* - p_i^*$, $\forall (i,j) \in \mathcal{E}$.

*Definition 2.3:* For a given $p^*$, the group solves *formation task w.r.t.* $p^*$ if all agents' trajectories asymptotically converge to the desired formation, i.e.,

$$\lim_{t \to \infty} \big(p_j(t) - p_i(t)\big) = p_j^* - p_i^* \qquad \forall (i,j) \in \mathcal{E}.$$

Furthermore, we assume that the graph is connected; in which case, the Laplacian matrix $L \in \mathbb{R}^{N \times N}$ is positive semidefinite and has an eigenvalue at zero with the corresponding eigenvector of all ones which is denoted by $\mathbb{1}$. $L$ is as well doubly stochastic, i.e., its row and column sum are zero.

In addition to the above distributed control tasks, we can add a group motion task where the group's centroid and orientation are controlled to achieve certain control behaviour, such as, following a given reference trajectory or converge to a desired point (e.g., group stabilization).

*Definition 2.4:* The group of $N$-agents is said to achieve *group stabilization* if the centroid of the group $p_{\text{cen}} := \frac{1}{N} \sum_i p_i$ converges asymptotically to the origin, i.e., $p_{\text{cen}}(t) \to 0$ as $t \to \infty$.

In the following section, we present our control design framework which can accomplish all three different control tasks. Both obstacle avoidance and formation tasks are solved by distributed control laws while group stabilization task is solved by a coordinated control law.

*Assumption 2.1:* At any given instantaneous time, at most one agent is within a given safe distance to an obstacle.

*Assumption 2.2:* For the group stabilization task, the formation centroid $p_{\text{cen}}$ is allowed to cross the obstacles.

## III. DISTRIBUTED OBSTACLE AVOIDANCE-FORMATION CONTROL DESIGN

The navigation function as used in [4] requires us to have apriori information of all tasks so that we can embed this information to the navigation function. In contrast to this approach, we will pursue a control design that is modular where one can add and remove control law for particular task directly without jeopardizing the completion of other tasks. Therefore we assume that the control law for each agent is a linear combination of control laws of different tasks, i.e.,

$$u_i = u_i^{\text{f}} + u_i^{\text{g}} + u_i^{\text{o}}, \quad i = 1, \ldots, N \tag{2}$$

where $u_i^{\text{f}}$ is the local control law for solving formation task of $i$-th agent, $u_i^{\text{g}}$ is the local control law for solving group stabilization task of $i$-th agent and $u_i^{\text{o}}$ is the local control law for solving collision avoidance task of $i$-th agent.

In the following sub-sections, we present a particular control law for each of the aforementioned tasks that will be used in our unified control framework. While our framework is not restricted to these control laws, we will focus mainly on these laws in this paper where we can demonstrate the applicability of our approach.

### A. Distributed formation control law

For achieving formation task, we consider the following standard relative position-based formation control law

$$u_i^{\text{f}} = c_{\text{f}} \sum_{j \in \mathcal{N}_i} w_{ij} \Big(p_j - p_i - \big(p_j^* - p_i^*\big)\Big), \tag{3}$$

where $c_{\text{f}} > 0$ is the formation gain and $w_{ij} > 0$, $i,j = 1, \ldots N$ the weight of the edge $(i,j) \in \mathcal{E}$. In compact form, the above law can be written as

$$U^{\text{f}} = c_{\text{f}}(L \otimes I_2)(p^* - p) \tag{4}$$

where $U^{\text{f}}$ is the stacked vector of $u_i^{\text{f}}$, $i = 1 \ldots N$ and $\otimes$ is the Kronecker product.

### B. Coordinated group stabilization control law

For stabilizing the group, we assume the existence of a central coordinator. The role of the central coordinator is to calculate/estimate the formation centroid $p_{\text{cen}}$ based on the position of the agents and use it to compute the stabilizing control law for the group.

The design of the control law for group stabilization is based on the assumption that the desired formation has been reached and there is no obstacle. In this case, using (1), (2), (3) and $u_i^o = 0$, the dynamics of the centroid's position is given by

$$\dot{p}_{\text{cen}} = \frac{1}{N} \sum_{i=1}^{N} \dot{p}_i = u^{\text{g}}, \tag{5}$$

where we assume that the control law $u^{\text{g}}$ will be communicated to all the agents and as such $u_1^{\text{g}} = u_2^{\text{g}} = \cdots = u_N^{\text{g}} = u^{\text{g}}$.

We propose the following group motion control law:

$$u^{\text{g}} = -c_{\text{P}} p_{\text{cen}} + c_{\text{I}} \gamma, \quad \dot{\gamma} = -p_{\text{cen}}, \tag{6}$$

where $c_{\text{P}} > 0$ and $c_{\text{I}} > 0$ are the proportional and integral gain, respectively. In compact form, we can write the group stabilization control law for the whole group as

$$U^{\text{g}} = \left( \mathbb{1}^{N \times 1} \otimes -c_{\text{P}} p_{\text{cen}} + c_{\text{I}} \gamma \right), \tag{7}$$

where $U^{\text{g}}$ is the stacked vector of $u_i^{\text{g}}$, $i = 1 \ldots N$.

If we do not need to solve obstacle avoidance task then the group stabilization task can be solved only by using the proportional controller. As it will be clear later, the integral action is needed in our control law for compensating the drift that is introduced by the distributed control law for the obstacle avoidance given in the next subsection.

We also remark that as each agent gets $u^{\text{g}}$, the formation control is not affected by the group motion control. Therefore, both control laws are complementary to each other.

### C. Distributed obstacle avoidance control law

In order to move safely towards the desired destination, the agents should avoid any obstacle during the course of transition. Since we have Assumption 2.1, at any given instantaneous time $t$, at most one agent can be in close vicinity of an obstacle. When an agent $i$ has the task of avoiding obstacle $\mathcal{O}_k$ (without considering the control law for other tasks), we can consider the following control law which has been proposed in [12]

$$u_i^o = c_\alpha \alpha_i^k(p_i) := \begin{cases} 0 & \text{if } \left\| p_i - p_{i,k}^* \right\| > R^{\text{safe}} \\ c_\alpha \frac{p_i - p_{i,k}^*}{\left\| p_i - p_{i,k}^* \right\|^2} & \text{if } \left\| p_i - p_{i,k}^* \right\| \leq R^{\text{safe}} \end{cases}, \tag{8}$$

where $c_\alpha > 0$ is the gain and

$$p_{i,k}^* := \operatorname*{argmin}_{x \in \partial \mathcal{O}_k} \operatorname{dist}(p_i, x). \tag{9}$$

So when the relative distance between an agent $i$ and the obstacle $k$ is less than the threshold $R^{\text{safe}}$, agent $i$ will activate the collision avoidance action. However, if we apply this obstacle avoidance control law locally then when it is activated locally on the $i$-th agent, the rest of the agents will only be driven by the formation and group

stabilization control laws (c.f. (2)). As a consequence, the unexpected obstacle avoidance manoeuvre by agent $i$ that is not communicated with the others will introduce undesirable deformation to the formation shape. On the other hand, the real-time communication of the obstacle avoidance control action to all nodes should be prohibited as it will unnecessarily consume the communication channel and is not scalable.

In order to 'diffuse' the obstacle avoidance action to the other agents, we introduce a dynamic obstacle avoidance controller whose state variable $\zeta_i$ is communicated to its neighbors. More precisely, the local dynamic obstacle avoidance controller is described by

$$\dot{\zeta}_i = c_\zeta \sum_{j \in \mathcal{N}_i} w_{ij}(\zeta_j - \zeta_i) + u_{\alpha_i} \tag{10}$$

$$u_i^o = \zeta_i, \tag{11}$$

where $c_\zeta > 0$ is the diffusion gain of obstacle avoidance control law, $w_{ij} > 0$, $i, j = 1, \ldots N$, is the weight of the edge $(i, j) \in \mathcal{E}$ and $u_{\alpha_i}$ is given by

$$u_{\alpha_i} = c_\alpha \alpha_i^k(p_i).$$

In compact form, the distributed dynamic obstacle avoidance control law is given by

$$\dot{\zeta} = -c_\zeta (L \otimes I_2) \zeta + U_\alpha$$
$$U^o = \zeta,$$

where $U^o$ is the stacked vector of $u_i^o$, $i = 1 \ldots N$, and respectively, $U_\alpha$ is the stacked vector of $u_{\alpha_i}$. The state variable $\zeta$ can be seen as an aggregation of the obstacle behavior of the individual agent.

If we pre-multiply (10) by $\frac{1}{N}(\mathbb{1}^\top \otimes I_2)$, we get

$$\frac{1}{N}(\mathbb{1}^\top \otimes I_2)\dot{\zeta} = \frac{1}{N}(\mathbb{1}^\top \otimes I_2)\left( -c_\zeta(L \otimes I_2)\zeta + U_\alpha \right)$$
$$\Rightarrow \dot{\zeta}_{\text{avg}} = \frac{1}{N} \sum_{i=1}^{N} u_{\alpha_i},$$

where $\zeta_{\text{avg}} := \frac{1}{N}(\mathbb{1}^\top \otimes I_2)\zeta$. When the agents are already free from obstacles after some finite time $\tilde{t} > 0$, then $U_\alpha(t) = 0$ for all $t > \tilde{t}$. In this case, the distributed obstacle avoidance control law becomes a consensus system which implies that $\zeta_i$ will converge to a common value given by $\frac{1}{N} \sum_j \zeta_j(\tilde{t})$ due to the average consensus protocol. Due to the integral action in the group stabilization controller, such constant bias from the asymptote of $\zeta_i(t)$ will be compensated and the integral controller ensures that the formation will not be deformed and the group's centroid converges to the origin.

## IV. NUMERICAL SIMULATION

### A. Simulation setup

To demonstrate the applicability of the proposed control law, we perform numerical simulations with a network of 3 agents.

As shown in Figures 1 and 2, we consider two circular obstacles in the plane with center at $p_{\text{obs}_1} = (4,7)$ and $p_{\text{obs}_2} = (1,4)$ (shown in red). The radii of the obstacles is set equally to $R_{\text{obs}} = 0.75$. For the obstacle avoidance control law, we take $R^{\text{safe}} = 0.5$. In these figures, the blue annulus shows the area within $R^{\text{safe}}$ from the obstacles where the obstacle avoidance control law is activated.

The gains of the control law are set to be $c_f = 10$, $c_P = 1$, $c_I = 0.9$, $c_\alpha = 400$, $c_\zeta = 10$.

The desired relative positions for the formation are set to be $p_{21}^* = [0, 3]^\top$, $p_{31}^* = [-2, 0]^\top$ in Figure 1 and $p_{21}^* = [-2, 2]^\top$ and $p_{31}^* = [2, 4]^\top$ in Figure 2.
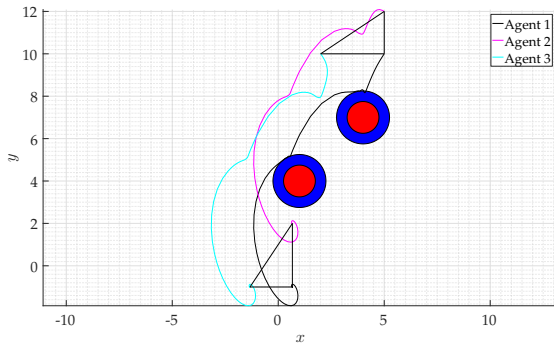


Fig. 1: State trajectory of three agents transitioning from initial position to final desired position; Agent 1 is within a distance $R^{\text{safe}}$ of both obstacles during the transition and avoids the obstacles.

*B. Simulation results*

We consider two initial conditions for simulating different interesting scenario. For the first one, during the group movement to the origin, there is only one agent that encounters both obstacles. For the second one, two of the agents must avoid the obstacles and the centroid pass through the obstacles.

The results of the closed-loop system using our proposed control law are shown in Figures 1 and 2. From these figures, we can observe that whenever an agent is within a distance $R^{\text{safe}}$ from the boundary of the obstacle (the blue region) then the obstacle avoidance behavior of that particular agent is 'activated'. As can also be seen in these figures, the distributed obstacle avoidance control law enables the diffusion of the avoidance manoevre to their neighbors. The neighboring agents undergo similar trajectories as that of the manoevring agent. The deformation of the group formation due to the collision avoidance is also minimized by the diffusive control law. Finally, one can observe that the group stabilization control law is able to steer the whole group towards the origin while compensating for the constant bias introduced by the distributed dynamic obstacle avoidance control law when they are exiting the blue region.
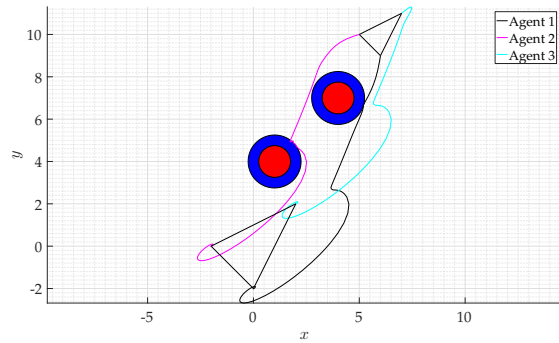


Fig. 2: State trajectory of three agents transitioning from initial position to final desired position. Agent 1 and Agent 2 avoids respectively obstacles $\mathcal{O}_1$ and $\mathcal{O}_2$ during transition while the centroid crosses obstacle $\mathcal{O}_1$.

## V. CONCLUSIONS

In this paper, we propose a new distributed control design law for achieving formation while avoiding obstacles for a group of agents. In combination with a coordinated group controller, we are able to steer the formation to the origin while maintaining formation shape during and after the obstacle avoidance manoevre.

### REFERENCES

[1] K.-K. Oh, M.-C. Park, and H.-S. Ahn, "A survey of multi-agent formation control," *Automatica*, vol. 53, pp. 424–440, Mar 2015.

[2] Q. Yang, M. Cao, H. Fang, and J. Chen, "Weighted centroid tracking control for multi-agent systems," in *2016 IEEE 55th Conference on Decision and Control (CDC)*. IEEE, Dec 2016.

[3] G. Antonelli, F. Arrichiello, F. Caccavale, and A. Marino, "Decentralized time-varying formation control for multi-robot systems," *The International Journal of Robotics Research*, vol. 33, no. 7, pp. 1029–1043, May 2014.

[4] Z. Kan, A. P. Dani, J. M. Shea, and W. E. Dixon, "Network connectivity preserving formation stabilization and obstacle avoidance via a decentralized controller," *IEEE Transactions on Automatic Control*, vol. 57, no. 7, pp. 1827–1832, Jul 2012.

[5] T. Mylvaganam, M. Sassano, and A. Astolfi, "A differential game approach to multi-agent collision avoidance," *IEEE Transactions on Automatic Control*, vol. 62, no. 8, pp. 4229–4235, Aug 2017.

[6] D. Panagou, D. M. Stipanovic, and P. G. Voulgaris, "Distributed coordination control for multi-robot networks using lyapunovlike barrier functions," *IEEE Transactions on Automatic Control*, vol. 61, no. 3, pp. 617–632, Mar 2016.

[7] M. Z. Romdlony and B. Jayawardhana, "Stabilization with guaranteed safety using control lyapunov–barrier function," *Automatica*, vol. 66, pp. 39–47, Apr 2016.

[8] A. D. Ames, X. Xu, J. W. Grizzle, and P. Tabuada, "Control barrier function based quadratic programs for safety critical systems," *IEEE Transactions on Automatic Control*, vol. 62, no. 8, pp. 3861–3876, Aug 2017.

[9] B. Anderson, C. Yu, B. Fidan, and J. Hendrickx, "Rigid graph control architectures for autonomous formations," *IEEE Control Systems Magazine*, vol. 28, p. 48–63, 2008.

[10] H. de Marina, B. Jayawardhana, and M. Cao, "Distributed rotational and translational maneuvering of rigid formations and their applications," *IEEE Transactions on Robotics*, vol. 32, no. 3, pp. 684–696, 2008.

[11] R. Olfati-Saber and R. Murray, "Graph rigidity and distributed formation stabilization of multi-vehicle systems," in *2002 IEEE 41st Conference on Decision and Control (CDC)*. IEEE, Dec 2002.

[12] K. P. Tee, S. S. Ge, and E. H. Tay, "Barrier lyapunov functions for the control of output-constrained nonlinear systems," *Automatica*, vol. 45, no. 4, p. 918–927, 2009.