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Damping, Inertia, and Delay Robustness Trade-offs in Power Systems

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***Abstract*— Electro-mechanical oscillations in power systems are typically controlled by simple decentralised controllers. We derive a formula for computing the delay margin of such controllers when the power system is represented by a simple mechanical network. This formula reveals a clear trade-off between system damping, inertia, and robustness to delays. In particular, it shows that reducing system inertia, which is a common consequence of increased renewable generation, can reduce robustness to unmodelled dynamics.**

I. Introduction

Electrical power systems are large, highly oscillatory, electro-mechanical networks. Electro-mechanical oscillations are typically controlled using simple, decentralised controllers, designed to improve damping based on physical insight [1]. However, as evidenced by the continued need to re-tune such devices in response to system changes, and that poorly tuned controllers have been a linked to several large blackouts [2], problems in this area clearly remain.

To further complicate matters, the introduction of renewable generation is causing power systems to change at an unprecedented rate. Although developments such as 'virtual inertia' allow these new devices to be operated in a similar fashion to conventional power system components, they are affecting the nature of the power system as a whole, by for example: **(a)** reducing system inertia; **(b)** increasing the loading of the transmission system; **(c)** introducing more delays and other dynamics. The main contribution of this paper is to derive a criterion that allow the effects of **(a)**–**(c)** to be quantified in terms of damping, inertia, and robustness.

The setting for this paper is the mechanical network in Figure 1. This is the well known mechanical analogue of the 'swing equation' power system model. The masses, which are constrained to lie on a circle, are analogous to the generators, and the springs to the transmission lines. This model obeys the differential equations

$$m_i\ddot{\theta}_i + \sum_{j=1}^{n} k_{ij}\sin\left(\theta_i - \theta_j\right) = f_i,\ i \in \{1,\ldots,n\}. \quad (1)$$

In the above $\theta_i(t)$ denotes the angular position of the *i*th mass on the circle, $m_i > 0$ its mass, and $f_i(t)$ an external force applied to it tangentially to the circle. The constants $k_{ij} \geq 0$ denote the spring constants of the spring connecting the *i*th and *j*th masses. In this paper we investigate trade-offs between damping, system inertia, and more complex

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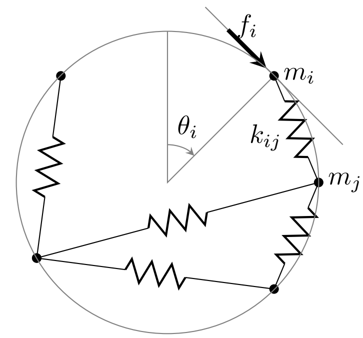

Fig. 1. Mechanical analogue of the swing equation model.

dynamics, when eq. (1) is impacted by forces of the form

$$f_i(t) = -d_i\left(\dot{\theta}_i(t-\tau_i) - \omega_0\right) + f_{0,i}. \quad (2)$$

In the above $d_i > 0$ is a damping constant, $\tau_i \geq 0$ a delay, $f_{0,i}$ some nominal force, and ω_0 a nominal angular velocity (analogous to the AC frequency in the power system).

The motivation for eq. (2) is that in the absence of delays, to a first approximation, most controllers for reducing electro-mechanical oscillations take this form. That is they draw insight from the mechanical analogue, and look to improve damping by applying forces proportional to deviation of the angular velocity $\dot{\theta}_i$ relative to some set-point (for example droop control, power system stabilisers, or resistive loads). The role of the delay is both to model an actual delay in the implementation of the control, and also to introduce more complex dynamics arising from unmodelled components. Therefore eqs. (1) and (2) provide a simple representation of a power system which captures the essence of **(a)**–**(c)**. The hope is that by better understanding the behaviour of this simple model, insight can be gained into how **(a)**–**(c)** will impact the behaviour of future power systems.

Our main contribution is to derive a formula for the 'delay margin' of eqs. (1) and (2) when linearised about an operating point. More specifically we give a formula for computing the largest number $\tau_{\max}$ such that the linearisation of eqs. (1) and (2) is stable for all $0 \leq \tau_i < \tau_{\max}$. This gives a measure of robustness to unmodelled dynamics. When the ratios $\frac{d_i}{m_i}$ are equal for all i, this expression takes the simple form

$$\tau_{\max} = \frac{\pi\left(\sqrt{\zeta^2+1}-\zeta\right)}{2\omega_p}. \quad (3)$$

Here ω_p is largest natural frequency of the linearisation of eq. (1), and ζ the damping ratio of this mode when the delay

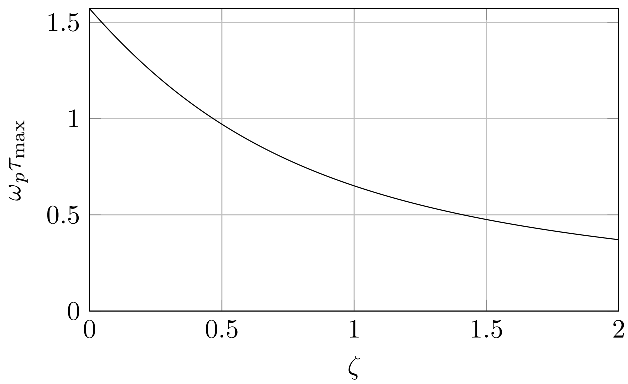

Fig. 2. Trade-off between the largest natural frequency of the system ω_p, the system-wide damping ratio ζ, and the delay margin $\tau_{\max}$.

equals zero. This formula, which is sketched in Figure 2, reveals a clear trade-off between damping, the delay margin, and the system natural frequencies.

Of particular significance with respect to **(a)–(c)** is that delay robustness decreases with an increase in the largest natural frequency. Since reducing the total inertia in the system will increase the largest natural frequency, we see that moving to a lower inertia system with more complex dynamics can significantly degrade robustness. Also of note is the inverse relationship between delay robustness and damping. This challenges the paradigm of using damping as an effective performance measure in a power system with increasingly complex dynamics, since it reveals that increasing damping will actually reduce delay robustness.

Notation

$A \succ (\succeq) 0$ denotes that a matrix is Hermitian and positive (semi-)definite. $A^{\frac{1}{2}}$ denotes the unique positive definite square root of $A \succ 0$, and $\sqrt{a}$ the positive square root of $a > 0$. $\exp(A)$ is the matrix exponential, and I the identity matrix. $\lambda_{\min}(A)$ is the smallest eigenvalue of a Hermitian matrix A. $\mathscr{H}_\infty$ is the space of transfer functions of stable linear, time-invariant, continuous time systems ($g(s) \in \mathscr{H}_\infty$ if it is analytic for $\mathrm{Re}\{s\} > 0$, and $\sup_{\mathrm{Re}\{s\}>0} |g(s)|$ is finite).

II. Results

In this section we state and prove a stability criterion for a delayed feedback interconnection of the form:

$$\begin{aligned} \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} &= \begin{bmatrix} 0 & Y^* \\ -Y & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 0 \\ Z^* \end{bmatrix} u \\ z &= \begin{bmatrix} 0 & Z \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \\ u_i &= -z_i (t - \tau_i), \quad i \in \{1, \ldots, m\}. \end{aligned} \tag{4}$$

Here $x(t) \in \mathbb{R}^p$ and $y(t) \in \mathbb{R}^n$, $n \leq p$, are the system states, $u(t), z(t) \in \mathbb{R}^m$ are the system input and output, respectively, $Z \in \mathbb{R}^{m \times n}$, and $Y \in \mathbb{R}^{n \times p}$.

We call such an interconnection stable if

$$\left(sI - \begin{bmatrix} 0 & Y^* \\ -Y & -Z^* \exp(-sT) Z \end{bmatrix} \right)^{-1} \in \mathscr{H}_\infty^{(n+p)\times(n+p)}. \tag{5}$$

where $T = \mathrm{diag}(\tau_1, \ldots, \tau_m)$. Amongst other things, this definition ensures that given any initial condition[1], the solution to eq. (4) satisfies $\lim_{t\to\infty}(x(t), y(t)) = 0$. Our main contribution is to show that the largest delays such that eq. (4) remains stable can be computed by solving an eigenvalue problem. In Section III we will show that this result can be used to calculate the delay margin of the swing equation model.

Theorem 1: Let $\tau \in \mathbb{R}$, $Y \in \mathbb{R}^{n\times p}, T \in \mathbb{R}^{m\times m}$, and $Z \in \mathbb{R}^{m\times n}$. If $\tau \geq 0$, $p \leq n$, $\mathrm{rank}(Y) = p$, and $\mathrm{rank}(Z) = n$, then the following statements are equivalent:

(i) For all T such that $0 \preceq T \preceq \tau I$, (5) holds.
(ii) $\tau < \tau_{\max}$, where

$$\tau_{\max} := -1/\lambda_{\min}\left(\frac{2}{\pi} \begin{bmatrix} 0 & Y^* \\ Y & -Z^* Z \end{bmatrix} \right).$$

Proof: The proof is structured as follows. We will first show that if (ii) is true, then (i) is true ((ii) $\Rightarrow$ (i)). We will then show that if (ii) is false, then (i) is false ($\neg$(ii) $\Rightarrow$ $\neg$(i)). This shows that (i) $\Rightarrow$ (ii) (proof by contrapositive), establishing the equivalence of the statements (i) and (ii).

((ii) $\Rightarrow$ (i)): It is sufficient to show that

$$\det\left(\begin{bmatrix} sI & -Y^* \\ Y & sI + Z^* \exp(-sT) Z \end{bmatrix} \right) \neq 0 \tag{6}$$

for all $s \in \mathbb{C}$ such that $\mathrm{Re}\{s\} \geq 0$, and for all $0 \preceq T \preceq \tau I$ where $\tau < \tau_{\max}$. Since

$$\det\left(\begin{bmatrix} 0 & -Y^* \\ Y & Z^* Z \end{bmatrix} \right) = \det(Z^* Z) \det\left(Y^* (Z^* Z)^{-1} Y \right),$$

which is greater than zero due to the rank conditions on Y and Z, there exists an ϵ such that eq. (6) holds for all $|s| \leq \epsilon$. We will now show that eq. (6) is invertible for the remaining values of s in the closed right half plane. Define

$$A(s) := \left(s^2 I + Y Y^* + s Z^* \exp(-sT) Z \right).$$

Applying the formula for evaluating blockwise determinants, it follows that

$$\det\left(\begin{bmatrix} sI & -Y^* \\ Y & sI + Z^* \exp(-sT) Z \end{bmatrix} \right) = s^{p-n} \det(A(s)).$$

Therefore it is sufficient to show that $A(s)$ is invertible for all $s \in \mathbb{C}$ such that $\mathrm{Re}\{s\} \geq 0$ and $|s| > \epsilon$. We will do this in three stages. We will first find an α such that $A(s)$ is invertible for all $|s| > \alpha$ and $T \succeq 0$. We will then show that subject to an upper bound on τ that depends on α, $A(s)$ is invertible for all $\epsilon \leq |s| \leq \alpha$. Finally we will show that (ii) implies the required bound on α in stage 2.

Stage 1: We will show that $A(s)$ is invertible for by showing that $|z^* A(s) z| > 0$ for all non-zero $z \in \mathbb{C}^n$. Since $T \succeq 0$, the matrix exponential $\exp(-sT)$ is a contraction

[1]Since eq. (4) contains a delay, the initial condition is some bounded function $(x_0, y_0) : [-\tau_{\max}, 0] \to \mathbb{R}^{p+n}$.

for all $\text{Re}\{s\} \geq 0$. This implies (by the Cauchy-Schwartz inequality) that for any $z \in \mathbb{C}^n$

$$|z^* Z^* \exp(-sT) Zz| \leq |z^* Z^* Zz|.$$

Furthermore

$$\left|z^* \left(s^2 I + YY^*\right) z\right| \geq \left|z^* \left(|s|^2 I - YY^*\right) z\right|.$$

Therefore by the reverse triangle inequality (and assuming that $|s|^2 I - YY^* \succeq 0$),

$$\begin{aligned}|z^* A(s) z| &\geq \left|\left|z^* \left(s^2 I + YY^*\right) z\right| - |z^* Z^* \exp(-sT) Zz|\right|, \\ &\geq z^* \left(|s|^2 I - YY^* - |s| Z^* Z\right) z.\end{aligned}$$

Therefore, if

$$|s|^2 I - YY^* - |s| Z^* Z \succ 0 \tag{7}$$

for all $|s| > \alpha$ (which also ensures that $|s|^2 I - YY^* \succeq 0$), then $A(s)$ is invertible for all $|s| > \alpha$. We will convert this into an eigenvalue problem. Applying the Schur complement lemma shows that eq. (7) is equivalent to

$$\begin{bmatrix} I & \frac{1}{|s|} Y^* \\ \frac{1}{|s|} Y & I - \frac{1}{|s|} Z^* Z \end{bmatrix} \succ 0.$$

Hence, if

$$\alpha := -\lambda_{\min}\left(\begin{bmatrix} 0 & Y^* \\ Y & -Z^* Z \end{bmatrix}\right), \tag{8}$$

then $A(s)$ is invertible for all $|s| > \alpha$.

Stage 2: We will show that $A(s)$ is invertible for $|s| \leq \alpha$ by showing that $\text{Im}\{z^* A(s) z\} > 0$ for all non-zero $z \in \mathbb{C}^n$ and $\text{Re}\{s\} \geq 0, \text{Im}\{s\} \geq 0$. This implies by conjugate symmetry that $\text{Im}\{z^* A(s) z\} < 0$ for all $z \neq 0, \text{Re}\{s\} \geq 0, \text{Im}\{s\} \leq 0$, implying invertibility for $\text{Re}\{s\} \geq 0$ as required. Since $\text{Im}\{s^2\} = 2\text{Re}\{s\}\,\text{Im}\{s\}$, it follows that

$$\text{Im}\left\{z^* \left(s^2 + YY^*\right) z\right\} \geq 0. \tag{9}$$

In addition, it is easily established that if

$$\text{Im}\{v^* s \exp(-s\Lambda) v\} > 0, \ \forall v \neq 0,$$

where Λ is the diagonal matrix of eigenvalues of T, then

$$\text{Im}\{z^* s Z^* \exp(-sT) Zz\} > 0, \ \forall z \neq 0.$$

It therefore follows that if $|s| T \preceq \frac{\pi}{2} I$, then for all $\text{Re}\{s\} \geq 0, \text{Im}\{s\} \geq 0, |s| > 0$,

$$\text{Im}\{z^* s Z^* \exp(-sT) Zz\} > 0, \ \forall z \neq 0. \tag{10}$$

From eqs. (9) and (10) it follows that if $\tau < \frac{\pi}{2\alpha}$, then $\text{Im}\{z^* A(s) z\} > 0$. Hence $A(s)$ is invertible for all $s \in \mathbb{C}$ such that $\text{Re}\{s\} \geq 0$ and $\epsilon < |s| \leq \alpha$.

Stage 3: We will show that (ii) implies that the requirements on α in stage 2 are fulfilled. To do so, we need only show that

$$\frac{\pi}{2\alpha} \leq \tau_{\max}.$$

This is immediate from the defintion of α in eq. (8).

($\neg$(ii) $\Rightarrow \neg$(i)): It is sufficient to show that (i) is false with $\tau \equiv \tau_{\max}$. Let $T = \tau_{\max} I$, which implies that $\alpha = \frac{\pi}{2\tau_{\max}}$. This implies that $\exp(-j\alpha T) = -jI$, and consequently that

$$A(j\alpha) = YY^* - \alpha^2 I + \alpha Z^* Z.$$

From eqs. (7) and (8) we see that this implies that there exists a $z \in \mathbb{C}^n$ such that $z^* A(j\alpha) z = 0$. Since $A(j\alpha)$ is Hermitian, this implies that $\det(A(j\alpha)) = 0$. Therefore for this T and s, eq. (6) is not invertible. Hence (i) is false as required. ■

III. Discussion

A. Applying Theorem 1 to the Swing Equations

In this section we will show how to use Theorem 1 to analyse the stability of the linearisation of the swing equation model in eqs. (1) and (2). Some care is required here, since the mechanical model in Figure 1 exhibits a rotational symmetry. This prevents us from speaking of 'stability' in the standard sense, since if $\theta_e \in \mathbb{R}^n$ is an equilibrium point, so is $\theta_e + c1_n$, for any constant c. We will resolve this using the standard trick of reducing the state dimension to eliminate this redundancy. To illustrate this, compactly write the linearisation of eqs. (1) and (2) about an equilibrium point as

$$\begin{aligned}\begin{bmatrix} M^{\frac{1}{2}} \dot{q} \\ M^{\frac{1}{2}} \ddot{q} \end{bmatrix} &= \begin{bmatrix} 0 & I \\ -M^{-\frac{1}{2}} L_B M^{-\frac{1}{2}} & 0 \end{bmatrix} \begin{bmatrix} M^{\frac{1}{2}} q \\ M^{\frac{1}{2}} \dot{q} \end{bmatrix} + \begin{bmatrix} 0 \\ M^{-\frac{1}{2}} u \end{bmatrix} \\ u_i &= -d_i \dot{q}_i(t - \tau_i).\end{aligned} \tag{11}$$

Here $q, \dot{q}, u$ are the values of $\theta, \dot{\theta}, f$ relative to equilibrium, $M = \text{diag}(m_1, \ldots, m_n)$, and L_B the weighted Laplacian matrix obtained from linearising the spring terms in eq. (1). To resolve the symmetry issue, we will find a new state x of lower dimension than q which has the 1_n mode removed. This means that the stability claims that follow do not imply that $\lim_{t\to\infty} q(t) = 0$, rather that

$$\lim_{t\to\infty} (q_i(t) - q_j(t)) = 0, \ i \neq j.$$

This means that the angles in between the springs settle to their equilibrium values, but the entire configuration may be rotated arbitrarily. In the process of performing this reduction, we will simultaneously put the linearisation in a form suitable for applying Theorem 1. We outline the relevant steps below.

The consequence of the equilibrium point not being unique is that $\text{rank}(L_B) < n$. We now make the following assumption:

Assumption 1: $L_B \succeq 0$.

This assumption allows $M^{-\frac{1}{2}} L_B M^{-\frac{1}{2}}$ to be factored as $QW_N^2 Q^*$, where $W_N \succ 0$ is diagonal, $Q \in \mathbb{R}^{n\times p}$ and $Q^* Q = I$. Hence we may rewrite eq. (11) as

$$\begin{aligned}\begin{bmatrix} W_N Q^* M^{\frac{1}{2}} \dot{q} \\ M^{\frac{1}{2}} \ddot{q} \end{bmatrix} &= \begin{bmatrix} 0 & W_N Q^* \\ -QW_N & 0 \end{bmatrix} \begin{bmatrix} W_N Q^* M^{\frac{1}{2}} q \\ M^{\frac{1}{2}} \dot{q} \end{bmatrix} + \begin{bmatrix} 0 \\ \tilde{u} \end{bmatrix} \\ \tilde{u}_i &= -\frac{d_i}{m_i}\left(\sqrt{m_i}\dot{q}_i(t - \tau_i)\right).\end{aligned}$$

Introducing the change of variables

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} W_N Q^* M^{\frac{1}{2}} & 0 \\ 0 & M^{\frac{1}{2}} \end{bmatrix} \begin{bmatrix} q \\ \dot{q} \end{bmatrix}$$

simplifies the above to the standard form

$$\begin{aligned} \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} &= \begin{bmatrix} 0 & W_N Q^* \\ -QW_N & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 0 \\ \tilde{u} \end{bmatrix} \\ \tilde{u}_i &= -\frac{d_i}{m_i} y_i (t - \tau_i). \end{aligned} \tag{12}$$

Equation (12) is of a suitable form for stability analysis with Theorem 1. Let

$$Z = \operatorname{diag}\left(\sqrt{\frac{d_1}{m_1}}, \ldots, \sqrt{\frac{d_n}{m_n}}\right) \text{ and } T = \operatorname{diag}(\tau_1, \ldots \tau_n).$$

Now putting eq. (12) into the form of eq. (4) shows that eq. (12) is stable if

$$\left(sI - \begin{bmatrix} 0 & W_N Q^* \\ -QW_N & -Z^* \exp(-sT) Z \end{bmatrix}\right)^{-1} \in \mathscr{H}_\infty^{(n+p)\times(n+p)}.$$

Since $T \succeq 0, p \leq n, \operatorname{rank}(QW_N) = p$ and $\operatorname{rank}(Z) = n$, Theorem 1 applies. Therefore the linearisation is stable for all

$$0 \leq \tau_i \leq \tau$$

if and only if

$$\tau < -1/\lambda_{\min}\left(\frac{2}{\pi}\begin{bmatrix} 0 & W_N Q^* \\ QW_N & -Z^* Z \end{bmatrix}\right).$$

B. Exhibiting Trade-offs using Theorem 1

In this section we derive eq. (3), which illustrated the trade-off discussed in the Introduction. In addition we show that similar relationships hold even in the fully heterogeneous case. To begin, assume that

$$\frac{d_1}{m_1} = \ldots = \frac{d_n}{m_n} =: \rho. \tag{13}$$

This simplification allows us to calculate $\tau_{\max}$ in terms of the natural frequencies of the undamped system and ρ. A number ω_i is a natural frequency of $M\ddot{q} + L_B q = f$, if there exists a $v \in \mathbb{C}^n$ such that

$$\omega_i^2 M v = L_B v.$$

The natural frequencies are therefore precisely the diagonal entries of the matrix W_N. Now, since

$$\begin{aligned} \det\left(\begin{bmatrix} sI & -W_N Q^* \\ -QW_N & sI + \rho I \end{bmatrix}\right) &= (s+\rho)^n \det\left(sI - \frac{1}{s+\rho} W_N^2\right) \\ &= (s+\rho)^{n-p} \prod_{i=1}^{p} \left(s^2 + \rho s - \omega_i^2\right), \end{aligned}$$

the eigenvalues of

$$\begin{bmatrix} 0 & W_N Q^* \\ QW_N & -\rho I \end{bmatrix}$$

are

$$\left\{-\rho, \frac{-\rho \pm \sqrt{\rho^2 + 4\omega_1^2}}{2}, \ldots, \frac{-\rho \pm \sqrt{\rho^2 + 4\omega_p^2}}{2}\right\}.$$

Consequently, assuming without loss of generality that ω_p is the largest natural frequency,

$$\tau_{\max} = \frac{\pi}{\rho + \sqrt{\rho^2 + 4\omega_p^2}} = \frac{\pi\left(\sqrt{\rho^2 + 4\omega_p^2} - \rho\right)}{4\omega_p^2}.$$

This expression is further simplified by defining the damping ratio[2] through the relationship $\rho = 2\zeta\omega_p$. More specifically the above becomes

$$\tau_{\max} = \frac{\pi\left(\sqrt{\zeta^2 + 1} - \zeta\right)}{2\omega_p},$$

which is precisely eq. (3). Similar trade-offs hold even if eq. (13) does not. To see this note observe that if $A = A^*$ and $B \succeq 0$, then

$$\lambda_{\min}(A) \geq \lambda_{\min}(A - B).$$

Therefore if $\rho_{\min} = \min_i \frac{d_i}{m_i}$ and $\rho_{\max} = \max_i \frac{d_i}{m_i}$, then

$$\frac{\pi\left(\sqrt{\zeta_{\max}^2 + 1} - \zeta_{\max}\right)}{2\omega_p} \leq \tau_{\max} \leq \frac{\pi\left(\sqrt{\zeta_{\min}^2 + 1} - \zeta_{\min}\right)}{2\omega_p},$$

where $\rho_{\max} = 2\zeta_{\max}\omega_p$ and $\rho_{\min} = 2\zeta_{\min}\omega_p$. Unlike before, calling $\zeta_{\min}$ and $\zeta_{\max}$ damping ratios is more suggestive than precise, but nevertheless indicates that a similar trade-off exits. The exact trade-off can always be found by solving the eignenvalue problem in Theorem 1.

C. Fundamental Performance Limitations

To avoid confusion, it is worth explicitly stating that the trade-offs we have discussed are a consequence of the particular form of eqs. (1) and (2). They certainly do not preclude the possibility that by using a more sophisticated controller, a better balance between damping, inertia, and robustness can be struck. The take home message is that by continuing to use simple controllers motivated by improving system damping in mechanical networks, stability issues may arise due to reduced system inertia or more complex unmodelled dynamics. Given the flexibililty of modern power electronic devices this presents a fantastic opportunity to the control community to exploit this design freedom to enhance performance in the power systems of the future.

IV. Conclusion

We showed that the delay robustness of a particular structured interconnection can be computed by solving an eigenvalue problem. This was used to derive a formula for the delay robustness of damping controllers in power networks. This formula revealed a clear trade-off between system damping, inertia, and delay robustness, illustrating that reducing inertia can significantly degrade robustness.

References

[1] G. Rogers, *Power System Oscillations*. Springer, 1999.
[2] V. Venkatasubramanian and Y. Li, “Analysis of 1996 Western American electric blackouts,” *Bulk Power System Dynamics and Control VI*, 2004.

[2]This definition is consistent with the scalar case, and equivalent to the damping ratios of the pole with natural frequency ω_p in eq. (12) if the delays equal zero.