23rd International Symposium on Mathematical Theory of Networks and Systems
Hong Kong University of Science and Technology, Hong Kong, July 16-20, 2018

Finite-Time Consensus Control for Second-order Multi-Agent Systems Based on Position Information

Zhenxing Li[1,2], Zhaodong Liu[2] and Guochen Pang[2]

Abstract— In this paper, we consider the distributed finite-time consensus control for a class of second-order nonlinear multi-agent systems. In order to design the finite-time consensus controller, homogeneous function was employed and a new type of distributed finite-time consensus controller was given. This kind of controller not only solves the finite-time consensus problem of leaderless second-order nonlinear multi-agent systems, but also relaxes the condition of the network topology among the agents from undirected graph or detail-balanced directed graph to strongly connected graph. In the real control engineering, the most common sensors are displacement sensor and acceleration sensor, the velocity information is not available directly. When velocity information is not available, the finite-time convergent observer is designed by using position information, and we give another kind of finite-time consensus controller for multi-agent systems based on the states of the finite-time observers.

I. INTRODUCTION

During the past decade, the distributed control of multi-agent systems has gained increasing attention in the system and control community for the broad range of applications. A typical control task of multi-agent systems is consensus, which is achieved by sharing information among the neighbourhood. When the states or outputs of all agents converge to a common value, we say that the consensus task of the multi-agent systems is achieved. A notable feature of consensus control is that the distributed controller only uses the local relative information rather than the full state information of multi-agent systems. Many effective coordination designs have been developed; see [1]-[3] and references therein.

Most of the existing coordination designs of multi-agent systems are asymptotical ones, which means that the multi-agent systems achieve control tasks in infinite settling time. However, under some practical situations, convergence rate of coordination protocols is an important issue. The finite-time consensus problems of first-order multi-agent systems were studied in [4]-[6]. The finite-time consensus, tracking and containment problems for second-order multi-agent systems were investigated in [7]-[11]. The literatures mentioned above only consider the first-order (or second-order) linear (or nonlinear) multi-agent systems. The studies of more general cases are seldom considered [12]-[13].

Most papers, mentioned above, are dealing with the distributed finite-time tracking control for leader-follower multi-agent systems with undirected network and the studies of finite-time consensus control for second-order nonlinear leaderless multi-agent systems are not reported. In this paper, we focus on the finite-time consensus control problems of a class of second-order nonlinear leaderless multi-agent systems with directed topology. Hence, we aim to solve the finite-time consensus problems for second-order nonlinear multi-agent systems by using the homogeneous system theory. When the velocity information is not available, we construct a finite-time convergent observer for each agent. Then, we use observers' states to design the finite-time consensus controller. We prove that the finite-time consensus problems of second-order nonlinear multi-agent systems can be solved by the proposed protocols.

II. Problem statement and preliminaries

The following notations will be used in the rest of this technical note. Let $\mathrm{Q}^{\mathrm{odd}}_{>0}$ be the set of all positive rational numbers with odd numerator and odd denominator, $\mathrm{Q}^{\mathrm{even}}_{>0}$ be the set of all positive rational numbers with even numerator and odd denominator.

A. Problem statement

In this paper, we consider a multi-agent system consisting of N agents. The dynamics of the ith $(i=1,\ldots,N)$ agent is described by

$$\begin{cases}\dot{p}_i = v_i,\\ \dot{v}_i = u_i + f(p_i, v_i),\end{cases} \tag{1}$$

where $p_i, v_i \in R$ are the position and velocity of the ith agent and u_i is the input of the ith agent, $f(\cdot)$ is a nonlinear continuous function.

The concerned finite-time consensus for leaderless multi-agent system (1) is given as:

Definition 1: The multi-agent system (1) is said to reach finite-time consensus, if for any initial condition, there exists a finite settling time T such that $\forall\ i,j = 1,\ldots,N$,

$$\lim_{t\to T}(p_i(t)-p_j(t))=0, \lim_{t\to T}(v_i(t)-v_j(t))=0,$$

and for all $t \geq T$

$$p_i(t)=p_j(t),\ v_i(t)=v_j(t).$$

*This work was supported by China Postdoctoral Science Foundation Grant under No.2017M620183, Jiangsu Planned Projects for Postdoctoral Research Funds under No.1701077B and Scientific Foundation for Doctor of Shandong province under No.ZR2017BF003.

[1]School of Mathematics, Southeast University Nanjing, 210096, China. Corresponding Author Email: zhxingli@gmail.com.

[2]School of Automation and Electrical Engineering and Key Laboratory of Complex systems and Intelligent Computing in Universities of Shandong(Linyi University), Linyi 276005, China

To facilitate the stability analysis of the closed-loop system, we make the following assumption on $f(\cdot)$.

Assumption 1: It is assumed that, for any $p_i, v_i, p_j, v_j \in R$, there exist $\theta > 0$ and $-\tau \in (0, 1/2) \cap \mathrm{Q}^{\mathrm{even}}_{>0}$ such that $f(\cdot)$ satisfies the following inequality:

$$|f(p_i, v_i) - f(p_j, v_j)| \leq \theta(|p_i - p_j|^{r_3/r_1} + |v_i - v_j|^{r_3/r_2}),$$

where $r_1 = 1,\ r_2 = 1 + \tau$ and $r_3 = 1 + 2\tau$.

B. Graph Theory

Throughout this technical note, we use a directed graph $\mathscr{G} = (\mathscr{V}, \mathscr{E}, A)$ to describe the information exchanged among agents, where $\mathscr{V} = \{1, 2, \ldots, N\}$ is the set of agents, $\mathscr{E} \subseteq \mathscr{V} \times \mathscr{V}$ is the set of edges and $A = [a_{ij}] \in R^{N \times N}$ is the associated adjacency matrix with weighing factors $a_{ij} \geq 0$. (i, j) denotes a directed edge from agent j to agent i. If $(i, j) \in \mathscr{E}$, $a_{ij} > 0$ and $a_{ij} = 0$ otherwise. A path from i_1 to i_l is a sequence of ordered edges of the form $(i_{k+1}, i_k) \in \mathscr{E},\ k = 1, \ldots, l-1$. A directed graph $\mathscr{G}$ is said to be strongly connected if there exists a directed path from every agent to every other agent.

III. Finite-time consensus control design

In this section, we consider the finite-time consensus problem of second-order nonlinear multi-agent system (1).

Denote $p_a = \sum_{i=1}^N \gamma_i p_i,\ \ v_a = \sum_{i=1}^N \gamma_i v_i$. Let $\bar{p}_i = p_i - p_a,\ \bar{v}_i = v_i - v_a$ be the position error and velocity error of the ith agent. Then, the error dynamics is given as follows:

$$\begin{cases} \dot{\bar{p}}_i = \bar{v}_i, \\ \dot{\bar{v}}_i = \bar{u}_i + \bar{f}_i(p, v),\ i = 1\ldots, N, \end{cases} \tag{2}$$

where $\bar{f}_i(p, v) = f(p_i, v_i) - \sum_{j=1}^N \gamma_j f(p_j, v_j)$, $\bar{u}_i = u_i - \sum_{j=1}^N \gamma_j u_j$. If the system (2) is finite-time stable at the equilibrium by a suitable feedback control, then the finite-time consensus problem of multi-agent system (1) is solved. Hence, we study the finite-time stabilization of system (2) rather than the finite-time consensus problem of multi-agent system (1). For convenience, we set

$$\begin{aligned} \tilde{p}_i &= \sum_{j=1}^N a_{ij}(p_i - p_j) = \sum_{j=1}^N a_{ij}(\bar{p}_i - \bar{p}_j), \\ \tilde{v}_i &= \sum_{j=1}^N a_{ij}(v_i - v_j) = \sum_{j=1}^N a_{ij}(\bar{v}_i - \bar{v}_j) \end{aligned} \tag{3}$$

as the cooperative errors, which will be used to construct the control input u_i.

For the convenience of design of controller and finite-time analysis, we introduce the following coordination transformation

$$\begin{aligned} &x_i = p_i/\kappa,\ y_i = v_i/\kappa, \\ &\bar{x}_i = \bar{p}_i/\kappa,\ \bar{y}_i = \bar{v}_i/\kappa, \\ &\tilde{x}_i = \tilde{p}_i/\kappa,\ \tilde{y}_i = \tilde{v}_i/\kappa,\ i = 1, \ldots, N, \end{aligned} \tag{4}$$

where $\kappa > 0$ is a constant to be determined later. Substituting new variables into multi-agent system (2) yields the following system

$$\begin{cases} \dot{\tilde{x}}_i = \tilde{y}_i \\ \dot{\tilde{y}}_i = \tilde{u}_i/\kappa + \tilde{f}_i(p, v)/\kappa,\ i = 1, \ldots, N. \end{cases} \tag{5}$$

Let $\beta_0, \beta_1 \in \mathrm{Q}^{\mathrm{odd}}_{>0}$ be the constants satisfying the following inequalities:

$$\beta_0 = r_2,\ (\beta_1 + 1)r_2 \geq (\beta_0 + 1)r_1 > 0. \tag{6}$$

It is easy to verify that $\beta_1 > 1$.

Theorem 1: For the second-order nonlinear multi-agent system (1), suppose the directed graph $\mathscr{G}$ is strongly connected and Assumption 1 holds, there exists constants $\kappa > 0$, $h_1 > 0$ and $h_2 > 0$ such that

$$u_i = -\kappa h_2 [\tilde{y}_i^{\beta_1} + (h_1 \tilde{x}_i^{r_2})^{\beta_1}]^{r_3/(\beta_1 r_2)} \tag{7}$$

solves the finite-time consensus problem of multi-agent system (1).

Proof: Consider the following Lyapunov function candidate

$$V = V_1 + \sum_{i=1}^N \gamma_i V_{i,2}, \tag{8}$$

where $V_1 = 2\tilde{x}^T \Gamma \tilde{x}^{r_2},\ V_{i,2} = \int_{u_{i,1}}^{\tilde{y}_i} [s^{\beta_1} - u_{i,1}^{\beta_1}] \mathrm{d}s$ with $u_{i,1} = -h_1 \tilde{x}_i^{r_2}$ and $h_1 = \frac{1}{1+r_2}$.

The time derivative of V along the trajectory of (??) and (7) is given by

$$\dot{V} = V' + + \sum_{i=1}^N \gamma_i \frac{\partial V_{i,2}}{\partial \tilde{y}_i} \frac{\tilde{f}_i(p, v)}{\kappa}, \tag{9}$$

where $V'(\tilde{x}, \tilde{y}) = \frac{1}{h_1}(\tilde{x}^T)^{r_2} \Gamma \tilde{y} + \sum_{i=1}^N \gamma_i \frac{\partial V_{i,2}}{\partial \tilde{x}_i} \tilde{y}_i + \sum_{i=1}^N \gamma_i \frac{\partial V_{i,2}}{\partial \tilde{y}_i} \tilde{u}'_i$ with $\tilde{u}'_i = \tilde{u}_i/\kappa$. Next, we will use two steps to complete the proof.

Step 1: We consider the first term of V'

$$\begin{aligned} &\frac{1}{h_1}(\tilde{x}^T)^{r_2} \Gamma \tilde{y} \\ &= -(\tilde{x}^T)^{r_2} \Gamma L \tilde{x}^{r_2} + \sum_{i=1}^N \frac{\partial V_1}{\partial \tilde{x}_i}(\tilde{y}_i - u_{i,1}) \\ &= -(\tilde{x}^T)^{r_2} \frac{(\Gamma L + L^T \Gamma)}{2} \tilde{x}^{r_2} + \sum_{i=1}^N \frac{\partial V_1}{\partial \tilde{x}_i}(\tilde{y}_i - u_{i,1}). \end{aligned} \tag{10}$$

By simple calculation, we obtain that $\sum_{i=1}^N \gamma_i \frac{\partial V_{i,2}}{\partial \tilde{y}_i} \tilde{u}'_i$ is nonpositive and negative when $\tilde{y}_i \neq u_{i,1}$. In the rest of Step 1, we will show that V' is negative definite by choosing a suitable h_2 under the following two cases.

Case 1: $\sum_{i=1}^N \gamma_i \frac{\partial V_{i,2}}{\partial \tilde{y}_i} \tilde{u}'_i = 0$.

In this case, $\tilde{y}_i = u_{i,1}, V_{i,2} = 0,\ \frac{\partial V_{i,2}}{\partial \tilde{x}_i} = 0$, and we have $V_{i,2} = 0$ for $i, j = 1, 2, \ldots, N$. Hence, for $\tilde{x} \neq 0, \tilde{y} \neq 0$, we have $V' = -(\tilde{x}^T)^{r_2}(\Gamma L + L^T \Gamma)\tilde{x}^{r_2} < 0$.

Case 2: $\sum_{i=1}^N \gamma_i \frac{\partial V_{i,2}}{\partial \tilde{y}_i} \tilde{u}'_i < 0$.

Define

$$
\begin{aligned}
S_+ &= \{[\tilde{x}^T, \tilde{y}^T]^T \in R^{2N} : V_0' \geq 0\}, \\
S_- &= \{[\tilde{x}^T, \tilde{y}^T]^T \in R^{2N} : V_0' < 0\}, \\
S_0 &= \{[\tilde{x}^T, \tilde{y}^T]^T \in R^{2N} : \Gamma(\tilde{x}, \tilde{y}) = 1\},
\end{aligned} \tag{11}
$$

where

$$
\begin{aligned}
V_0' &= \frac{2}{h_1}(\tilde{x}^T)^{r_2}\Gamma L \tilde{y} + \sum_{i=1}^{N} \gamma_i \frac{\partial V_{i,2}}{\partial \tilde{x}_i}\tilde{y}_i, \\
\Gamma(\tilde{x}, \tilde{y}) &= \left(\sum_{i=1}^{N} \tilde{x}_i^{c/r_1} + \sum_{i=1}^{N} \tilde{y}_i^{c/r_2}\right)^{1/c}
\end{aligned} \tag{12}
$$

with $c > 1$. We suppose S_+ is nonempty. Set

$$
M_1 \overset{\text{def}}{=} \max_{[\tilde{x}^T, \tilde{y}^T]^T \in S_0} V_0', \ M_2 \overset{\text{def}}{=} \min_{[\tilde{x}, \tilde{y}]^T \in S_0 \cap S_+} - \sum_{i=1}^{N} \gamma_i \frac{\partial V_{i,2}}{\partial \tilde{y}_i} \frac{\tilde{u}_i'}{h_2}.
$$

From equation (7), we learn that $\tilde{u}_i'/h_2$ does not depend on h_2. Take $h_2 > M_1/M_2$, then $V' < 0$ for all $[\tilde{x}^T, \tilde{y}^T]^T \in S_0$.

For $\varepsilon = \Gamma(\tilde{x}, \tilde{y}) > 0$, let $\varepsilon = \Gamma(\tilde{x}, \tilde{y}) > 0$ and

$$
[\check{x}^T, \check{y}^T]^T = (\tilde{x}_1/\varepsilon, \ldots, \tilde{x}_N/\varepsilon, \tilde{y}_1/\varepsilon^{r_2}, \ldots, \tilde{y}_N/\varepsilon^{r_2})^T \in S_0.
$$

Because of homogeneity of $V'(\tilde{x}, \tilde{y})$, for any $[\tilde{x}^T, \tilde{y}^T]^T \neq 0$, we get

$$
V'(\tilde{x}, \tilde{y}) = \varepsilon^{(\beta_1+1)r_2+\tau} V'(\check{x}, \check{y}) < 0,
$$

and we have

$$
V'(\tilde{x}, \tilde{y}) \leq K_1 V(\tilde{x}, \tilde{y})^{\frac{(\beta_1+1)r_2+\tau}{(\beta_1+1)r_2}}, \tag{13}
$$

where $K_1 = \max\limits_{\{[\tilde{x}^T, \tilde{y}^T]^T : V(\tilde{x}, \tilde{y}) = 1\}} V'(\tilde{x}, \tilde{y}) < 0$.

Step 2: Now, we consider $\sum_{i=1}^{N} \gamma_i \frac{\partial V_{i,2}}{\partial \tilde{y}_i} \frac{\tilde{f}_i(p,v)}{\kappa}$. Under Assumption 1, we can prove that

$$
\dot{V}(\tilde{x}, \tilde{y}) \leq V'(\tilde{x}, \tilde{y}) - \theta\rho\sigma\kappa^{\tau} K_2 V'(\tilde{x}, \tilde{y}). \tag{14}
$$

where $\rho > 0, \sigma > 0$ and $K_2 > 0$ are known constants. Choosing a large enough κ satisfying $1 - \theta\rho\sigma\kappa^{\tau} K_2 > 0$, the right side of inequality (14) is negative definite. In a similar manner as inequality (13), the above inequality can be rewritten as

$$
\dot{V} \leq (1 - \theta\kappa^{\tau} K_2) K_1 V^{\frac{(\beta_1+1)r_2+\tau}{(\beta_1+1)r_2}} \tag{15}
$$

which implies that the finite-time consensus problem of multi-agent system (1) is solved. ■

IV. Finite-time consensus controller based on position information

In this section, we consider the finite-time consensus problem of leaderless multi-agent systems (1) when the velocity information is not available. For each agent, a finite-time observer is constructed to estimate the unknown states. Then, we use the estimated states to design the distributed finite-time consensus controller u_i.

The finite-time observer for agent i is designed as [14]?

$$
\begin{cases}
\dot{\hat{p}}_i = \hat{v}_i + k_1 H (p_i - \hat{p}_i)^{r_2}, \\
\dot{\hat{v}}_i = u_i + f(p_i, \hat{v}_i) + k_2 H^2 (p_i - \hat{p}_i)^{r_3}, \ i = 1, \ldots, N,
\end{cases} \tag{16}
$$

where k_1, k_2 and $H > 1$ are constants. Denote

$$
\begin{aligned}
\hat{x}_i &= \sum_{j=1}^{N} a_{ij} \frac{\hat{p}_i - \hat{p}_j}{\kappa}, \quad \hat{y}_i = \sum_{j=1}^{N} a_{ij} \frac{\hat{v}_i - \hat{v}_j}{\kappa}, \\
\hat{x} &= [\hat{x}_1, \ldots, \hat{x}_N]^T, \qquad \hat{y} = [\hat{y}_1, \ldots, \hat{y}_N]^T,
\end{aligned}
$$

where κ is the constant determined in Theorem 1. We give the distributed finite-time consensus controller as follows:

$$
u_i(\hat{x}, \hat{y}) = -\kappa h_2 [\hat{y}_i^{\beta_1} + (h_1 \hat{x}_i^{r_2})^{\beta_1}]^{r_3/(\beta_1 r_2)}. \tag{17}
$$

Theorem 2: For multi-agent system (1), suppose the directed graph $\mathscr{G}$ is strongly connected and Assumption 1 holds, then distributed controller (17) based on observers (16) solves the finite-time consensus problem of multi-agent system (1).

Proof: Denote $e_{i,p} = p_i - \hat{p}_i, e_{i,v} = v_i - \hat{v}_i, i = 1, \ldots, N$, as the observer errors. Set $e_p = [e_{1,p}, \ldots, e_{N,p}]^T$, $e_v = [e_{1,v}, \ldots, e_{N,v}]$, $e = [e_p^T, e_v^T]^T$. We can get the following system

$$
\begin{cases}
\dot{e}_{i.p} = H e_{i,v} - k_1 H (e_{i,p})^{r_2}, \\
\dot{e}_{i,v} = -k_2 H^2 (e_{i,p})^{r_3} + \dfrac{f(p_i, v_i) - f(p_i, \hat{v}_i)}{H}.
\end{cases} \tag{18}
$$

By [14], there exist k_1, k_2 and $H \geq 1$ such that the error system (18) is globe finite-time stable. And there exist a Lyapunov function $W_i(e_i)$ and a positive function $\Psi_i(e_i)$ for system (18) satisfying $\dot{W}_i(e_i) \leq -\Psi_i(e_i)$. For the augmented system (5) and (18), we choose the following Lyapunov function

$$
V(\tilde{x}, \tilde{y}, e) = V^2(\tilde{x}, \tilde{y}) + K \sum_{i=1}^{N} W_i^{\alpha}(e_i), \tag{19}
$$

where $\alpha = (\beta_1 + 1) r_2 > 1$, $K > 0$ is large number to be determined. Take the derivative of $V(\tilde{x}, \tilde{y}, e)$ along (5) and (18), we get

$$
\dot{V}(\tilde{x}, \tilde{y}, e) \leq -B_1(\tilde{x}, \tilde{y}) - K B_2(e) - B_3(\tilde{x}, \tilde{y}, e), \tag{20}
$$

where

$$
\begin{aligned}
B_1(\tilde{x}, \tilde{y}) &= -2K_1 V^{\frac{2\alpha - \tau}{\alpha}}(\tilde{x}, \tilde{y}), \\
B_2(\mathbf{e}) &= \sum_{i=1}^{N} \alpha W_i^{\alpha-1}(e_i) \cdot \Psi_i(e_i), \\
B_3(\tilde{x}, \tilde{y}, e) &= \sum_{i=1}^{N} 2V(\tilde{x}, \tilde{y}) \cdot \frac{\partial V(\tilde{x}, \tilde{y})}{\partial \tilde{y}_i} [\tilde{u}_i(\tilde{x}, \tilde{y}) - \tilde{u}_i(\hat{x}, \hat{y}, e)].
\end{aligned}
$$

Like the proof of Theorem 1, we can prove that

$$
\dot{V}(\tilde{x}, \tilde{y}, e) \leq C V^{\frac{2\alpha+\tau}{2\alpha}}(\tilde{x}, \tilde{y}, e), \tag{21}
$$

where $C = -\min\limits_{\{(\tilde{x}^T, \tilde{y}^T, e) : V(\tilde{x}, \tilde{y}, e) = 1\}} B(\tilde{x}, \tilde{y}, e) < 0$. ■

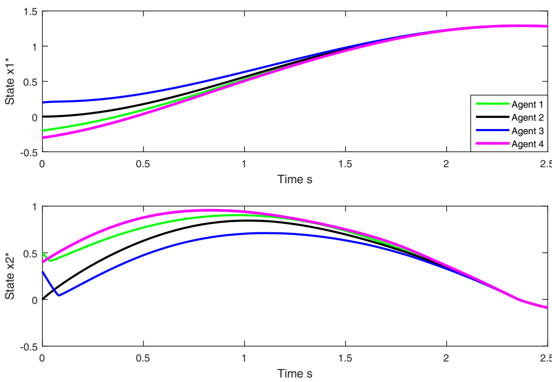

Fig. 1. Trajectories of multi-agent system (22) with controller (7)

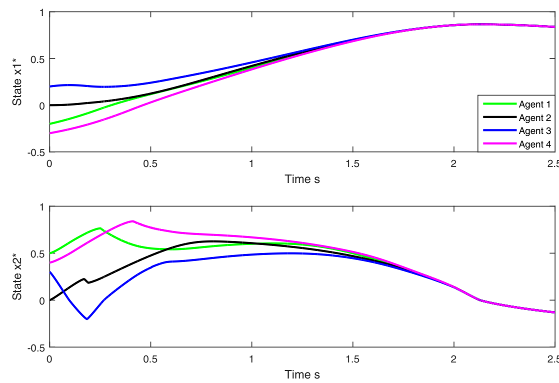

Fig. 2. Trajectories of multi-agent system (22) with controller (17)

V. Simulations

In this section, we use an example to illustrate the proposed finite-time consensus and protocol (7). We consider a multi-agent system with four agents labeled as 1, 2, 3, 4. The dynamics of ith agent is described by

$$\begin{cases} \dot{p}_i = v_i, \\ \dot{v}_i = u_i - \sin(p_i) - v_i^{1/3}, \ i = 1, \ldots, 4. \end{cases} \tag{22}$$

It is easy to verify that $f(p_i, v_i) = -\sin(p_i) - v_i^{1/3}$ satisfies Assumption 1 with $\theta = 2$ and $\tau = -\frac{2}{5}$. The Laplacian matrix of graph $\mathscr{G}$ is given as follows:

$$L = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ -1 & 0 & 2 & -1 \\ -1 & 0 & 0 & 1 \end{bmatrix}.$$

We design the consensus controller (7) for agents 1, 2, 3, 4 to reach finite-time consensus. According to Theorem 1, we set $r_1 = 1, r_2 = \frac{3}{5}, \beta_0 = \frac{3}{5}, \beta_1 = 2$ and $\kappa = 5$. Under controller (7), the response curves of agents are given in Fig. 1.

When the velocity information is not available, we use the following finite-time observer to estimate the state of agent i:

$$\begin{cases} \dot{\hat{p}}_i = \hat{v}_i + k_1 H (p_i - \hat{p}_i)^{r_2}, \\ \dot{\hat{v}}_i = u_i + \sin(p_i) - \hat{v}_i^{1/3} + k_2 H^2 (p_i - \hat{p}_i)^{r_3}, \end{cases} \tag{23}$$

where $k_1 = 4$, $k_2 = 1$, $H = 2$, $r_3 = 1/5$, $\hat{p}_i(0) = 0$ and $\hat{v}_i(0) = 0$. Then, we design finite-time consensus controller (17), and the associated response curves of agents are given in Fig. 2.

VI. Conclusions

This paper has studied the finite-time consensus problems of second-order nonlinear leaderless multi-agent systems with directed topology. Based on the theories of finite-time control and homogeneous systems, we give two kinds of finite-time consensus protocols. We prove that the finite-time control objectives can be achieved by the proposed protocols. Finally, a simulation is given to illustrate the effectiveness of the proposed protocol.

References

[1] A. Jabdabaie, J. Lin, and S. M. Morse, "Coordination of groups of mobile autonomous agents using nearest neighbor rules," IEEE Trans. Autom. Control, vol. 48, no. 6, pp. 988-1001, Jun. 2003.

[2] H. Su, X. Wang, and Z. Lin, "Flocking of multi-agents with a virtual leader," IEEE Trans. Autom. Control, vol. 54, no. 2, pp. 293-307, Feb. 2009.

[3] H. Zhang and F. Lewis, "Adaptive cooperative tracking control of high-order nonlinear systems with unknown dynamics," Automatica, vol. 47, no. 7, pp. 1432-1439, Jul. 2012.

[4] J. Cortés, "Finite-time convergent gradient flows with applications to network consensus," Automatica, vol. 42, no. 11, pp. 1993-2000, Nov. 2006.

[5] Q. Hui, W. M. Haddad, and S. P. Bhat, "Finite-time semistability and consensus for nonlinear dynamical networks," IEEE Trans. Autom. Control, vol. 53, no. 8, pp. 1887-1990, Sep. 2008.

[6] F. Xiao, L. Wang, J. Chen, and Y. Gao, "Finite-time formation control for multi-agent systems," Automatica, vol. 45, no. 11, pp. 2605-2611, Nov. 2009.

[7] X. He, Q. Wang and W. Yu, "Finite-time containment control for second-order multiagent systems under directed topology," IEEE Trans. Circuits and Systems II, vol. 61, no. 8, pp. 619-623, Aug. 2014.

[8] X. Wang and Y. Hong, "Finite-time consensus for multi-agent networks with second-order agent dynamics," in Proceedings of the 17th World Congress The International Federation of Automatic Control, pp. 15185-15190, Seoul, Korea, Jul 6-11, 2008.

[9] Z. Guan, F. Sun, Y. Wang, and T. Li, "Finite-time Consensus for leader-following second-order multi-agent networks," IEEE Trans. Circuits. Syst., vol. 59, no. 11, pp. 2646-2654, Nov. 2012.

[10] X. Wang, S. Li, and P. Shi, "Distributed finite-time containment control for double-integrator multiagent systems," IEEE Trans. Cybern., vol. 61, no. 6, pp. 1778-1788, Sep. 2014.

[11] N. Zhou, Y. Xia, M. Wang, and M. Fu, "Finite-time attitude control of multiple rigid spacecraft using terminal sliding mode," Int. J. Robust Nonlinear Control, vol. 25, pp. 1862-1876, Apr. 2014.

[12] H. Yu, Y. Shen, and X. Xia, "Adaptive finite-time consensus in multi-agent networks," Syst. Control Lett., vol. 62, no. 10, pp. 880-889, Oct. 2013.

[13] Y. Zhou, X. Yu, C. Sun, and W. Yu, "Higher order finite-time consensus protocol for heterogeneous multi-agent systems," Int. J. Control, vol. 88, no. 2, pp. 285-294, Apr. 2015.

[14] H. Du, C. Qian, S. Yang, and S. Li, "Recursive design of finite-time convergent observers for a class of time-varying nonlinear systems," Automatica, vol. 49, no. 2, pp. 601-609, Feb. 2013.